

Izvod funkcije, obnavljanje

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1 Zadaci

1. $y = \frac{\arccotgx}{x}$
2. $y = \sqrt{1+x^4}$
3. $y = \sqrt[4]{1-x^2}$
4. $y = \frac{1+x\sqrt{x}}{1-x\sqrt{x}}$
5. $y = \frac{e^x+e^{-x}}{e^x-e^{-x}}$
6. $y = (x-1)^3(x+1)$
7. $y = \arccotg\frac{1+x}{1-x}$
8. $y = tg\frac{1+x}{1-x}$
9. $y = \arcsin(\cos 2x)$
10. $y = \ln \frac{x+\sqrt{1-x^2}}{x}$

2 Rešenja

1. $y = \frac{\arccotgx}{x}$
$$y' = \frac{(\arccotgx)'x - x'\arccotgx}{x^2} = \frac{-\frac{1}{1+x^2}x - \arccotgx}{x^2} = -\frac{1}{x(1+x^2)} - \frac{\arccotgx}{x^2}$$
2. $y = \sqrt{1+x^4}$
$$y' = \frac{1}{2\sqrt{1+x^4}}(1+x^4)' = \frac{2x^3}{\sqrt{1+x^4}}$$
3. $y = \sqrt[4]{1-x^2}$
$$y' = \frac{1}{4}(1-x^2)^{-\frac{3}{4}}(1-x^2)' = -\frac{x}{2\sqrt[4]{(1-x^2)^3}}$$

$$4. \ y = \frac{1+x\sqrt{x}}{1-x\sqrt{x}}$$

$$y' = \frac{(1+x\sqrt{x})'(1-x\sqrt{x}) - (1+x\sqrt{x})(1-x\sqrt{x})'}{(1-x\sqrt{x})^2} = \frac{\frac{3}{2}\sqrt{x}(1-x\sqrt{x}) - (1+x\sqrt{x}) \cdot (-\frac{3}{2})\sqrt{x}}{(1-x\sqrt{x})^2} = \frac{3\sqrt{x}}{(1-x\sqrt{x})^2}$$

$$5. \ y = \frac{e^x + e^{-x}}{e^x - e^{-x}}$$

- $y' = \frac{(e^x + e^{-x})'(e^x - e^{-x}) - (e^x + e^{-x})(e^x - e^{-x})'}{(e^x - e^{-x})^2} = \frac{(e^x e^{-x})^2 - (e^x + e^{-x})^2}{(e^x - e^{-x})^2} = -\frac{4}{(e^x - e^{-x})^2}$
- $\sinh x = \frac{e^x - e^{-x}}{2}, \cosh x = \frac{e^x + e^{-x}}{2}, \operatorname{ctgh} x = \frac{e^x + e^{-x}}{e^x - e^{-x}} = y, (\operatorname{ctgh} x)' = -\operatorname{csech}^2 x$
 $y' = (\operatorname{ctgh} x)' = -\frac{1}{\left(\frac{e^x - e^{-x}}{2}\right)^2} = -\frac{4}{(e^x - e^{-x})^2}$

$$6. \ y = (x-1)^3(x+1)$$

$$y' = ((x-1)^3)'(x+1) + (x-1)^3(x+1)' = 3(x-1)^2(x-1)'(x+1) + (x-1)^3 = 2(x-1)^2(2x+1)$$

$$7. \ y = \operatorname{arccotg} \frac{1+x}{1-x}$$

$$y' = -\frac{1}{1+\left(\frac{1+x}{1-x}\right)^2} \left(\frac{1+x}{1-x}\right)' = -\frac{(x-1)^2}{2(x^2+1)} \frac{(1+x)'(1-x) - (1+x)(1-x)'}{(1-x)^2} = -\frac{1}{1+x^2} = \operatorname{arccotg}' x$$

$$8. \ y = \operatorname{tg} \frac{1+x}{1-x}$$

$$y' = \frac{1}{\cos^2 \frac{1+x}{1-x}} \left(\frac{1+x}{1-x}\right)' = \frac{2}{(1-x^2)\cos^2 \frac{1+x}{1-x}}$$

$$9. \ y = \operatorname{arcsin}(\cos 2x)$$

$$y' = \frac{1}{\sqrt{1-\cos^2 2x}} (\cos 2x)' = \frac{1}{|\sin 2x|} \cdot (-2)\sin 2x = -2\operatorname{sgn}(\sin 2x)$$

$$10. \ y = \ln \frac{x+\sqrt{1-x^2}}{x}, \ln f(x) = \frac{f'(x)}{f(x)}$$

$$y' = \frac{\left(\frac{x+\sqrt{1-x^2}}{x}\right)'}{\frac{x+\sqrt{1-x^2}}{x}} = \frac{\frac{(x+\sqrt{1-x^2})'x - (x+\sqrt{1-x^2})x'}{x^2}}{\frac{x+\sqrt{1-x^2}}{x}} = \frac{x\left(1+\frac{1}{2\sqrt{1-x^2}}(1-x^2)'\right) - (x+\sqrt{1-x^2})}{x(x+\sqrt{1-x^2})} =$$

$$= -\frac{1}{x\sqrt{1-x^2}(x+\sqrt{1-x^2})}$$